A New Method for the Time-Domain Analysis of Lossy Coupled Transmission Lines

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Abstract– A new method, based on numerical inversion of the Laplace transform, is presented for the analysis of lossy coupled transmission lines with arbitrary linear terminal and interconnecting networks. The method is more reliable and efficient than previously published techniques based on the fast Fourier transform.

I. Introduction

Evaluation of the time domain response of multiconductor transmission lines is of great importance in the characterization of high speed interconnections frequently encountered in the design of digital computers and communication systems. Improperly designed interconnects can result in increased signal delay because of losses, inadvertent switching and noise because of crosstalk, false switching and ringing due to reflections[1, 2]. This phenomena can be observed at both the chip and system levels where the interconnected blocks can be analog, digital or a combination of both. With subnanosecond rise times, the electrical length of interconnects can become a significant fraction of a wavelength. Consequently the conventional lumped-impedance interconnect model is not adequate in this case. Instead, a distributed transmission line model should be used.

Several techniques have been proposed in the literature for the analysis of coupled microstrip lines[3, 4, 5, 6]. The common method[6] for analyzing the time domain response of a lossy multiconductor transmission line terminated by an arbitrary linear network is based on separate formulation for the equations describing the transmission lines and the equations describing the terminal and interconnecting networks. These equations are combined at the analysis stage. The analysis is performed in the frequency domain at a set of discrete frequencies. Time domain waveforms are obtained using the inverse fast Fourier transform. This approach has a major difficulty when the analysis has to span a time interval of several line transient times. For example the response of a lossless line with short-circuited ports is of infinite duration. Consequently, it is impossible in this case to compute the response using FFT. Even for moderately lossy lines, the duration of the response exceeds many transit times of the transmission line network. This makes the use of inverse FFT techniques inefficient as a large number of points must be added to the analysis to avoid aliasing problems. An important application in which such situations arise is the analysis of lossy multiconductor transmission lines with arbitrary nonlinear terminations [7, 8]. In addition, the absence of a simple error criterion for the inverse FFT makes it difficult to establish confidence in the results obtained in this case.

A new method for analyzing the time domain response of linear transmission line networks is presented. This new method, based on the modified nodal admittance(MNA) matrix[9], unifies the formulation of the network equations including the coupled transmission lines, terminal and interconnecting networks. The method uses numerical inversion of the Laplace transform as an alternative to the FFT to obtain the time domain solutions. In the following sections, the advantages of the proposed technique are discussed.

II. Formulation of the Network Equations

The formulation of the network equations is based on the modified nodal admittance matrix. Each multiterminal component in the linear network has a "stamp" which defines the corresponding entries in the MNA matrix. These entries relate the current and voltage transforms at terminals of the linear component. Details of the MNA formulation and examples of the stamps for linear components can be found in [10, 9].

The MNA stamp for the lossy multiconductor transmission line is an admittance representation in the form $[Y][V] = [I]$, where $[Y]$ is a matrix of admittance parameters, $[V]$ is the matrix of terminal voltages and $[I]$ is the matrix of terminal currents. The stamp is developed assuming a transmission line uniform along its length with an arbitrary cross section. The cross section, with N signal conductors and a reference, can be represented by the following $N \times N$ matrices of line parameters: the inductance per unit length $[L]$, the resistance per unit length $[R]$, the capacitance per unit length $[B]$, and the conductance per

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unit length $[G]$. The MNA stamp for the lossy multiconductor transmission line can be shown to be[11]

$$[Y] = \begin{bmatrix} S_i E_1 S_v^{-1} & S_i E_2 S_v^{-1} \\ S_i E_2 S_v^{-1} & S_i E_1 S_v^{-1} \end{bmatrix} \quad (1)$$

where $[E_1]$ and $[E_2]$ are diagonal matrices

$$[E_1] = diagonal\left\{\frac{\exp(-2\gamma_m D)+1}{1-\exp(-2\gamma_m D)}, m=1,N\right\}$$

$$[E_2] = diagonal\left\{\frac{2}{\exp(-\gamma_m D)-\exp(\gamma_m D)}, m=1,N\right\}$$

D is the line length, γ_m^2 and $[S_v]$ are the eigenvalues and eigenvectors of the wave equation

$$\det\left\{\gamma_m^2[U]-[Z_p][Y_p]\right\}=0 \quad (2)$$

and

$$[Z_p]=[R]+s[L], \quad [Y_p]=[G]+s[C] \quad (3)$$

$$[S_i]=[Z_p]^{-1}[S_v][\Gamma] \quad (4)$$

$[U]$ is an identity matrix and $[\Gamma]$ is a diagonal matrix $\{\gamma_1, \gamma_2, ..., \gamma_N\}$.

Using the MNA stamp for the lossy coupled transmission line and the stamps for all of the other components of the network a MNA formulation $[Y(s)][V(s)] = [I(s)]$ representing the network in the Laplace domain is obtained. The frequency response of the network is obtained with $s = jw$. The time domain response is obtained directly using numerical inversion of the Laplace transform as described in the next section.

III. Computation of Time domain Response using Numerical Inversion of the Laplace Transform

The method for numerical inversion of the Laplace transform is described in [10, 12]. The method involves the computation of the frequency domain function at preassigned complex points and forming a weighted sum. It exactly inverts a certain number of terms of the Taylor series expansion of the time response and is thus equivalent to the methods used for the integration of differential equations. It has been shown that the method is absolutely stable and that the equivalent order of integration can be changed between 1 and 46 without affecting the stability properties.

Given a Laplace domain MNA representation for a network containing lossy multiconductor transmission lines, $[Y(s)][V(s)] = [I(s)]$ where $[V(s)]$ is the circuit response, the time domain response is calculated using numerical inversion of the Laplace transform as

$$[\hat{v}(t)] = -(1/t)\sum_{i=1}^{M'} \mathbf{Re}\left[K_i'[Y(z_i/t)]^{-1}[I(z_i/t)]\right] \quad (5)$$

The response $[\hat{v}(t)]$ at each time point is obtained from the M' solutions to the network equation $[Y(s)]^{-1}[I(s)]$ evaluated at the complex frequencies $s = z_i/t$. Values for K_i' and z_i vary with M', the order of the approximation, and are available in tables[10].

Several points of interest should be mentioned when comparing (5) with inverse FFT methods. First, the numerical inversion of the Laplace transform does not suffer from aliasing effects. Second, the number of frequency points defined by $M' < 15$ is usually much less than the number of frequency points required by the inverse FFT algorithm for the same accuracy in the output waveforms. This leads to considerable savings in the CPU requirements. Third, from (5), it is obvious that the solution at time t is completely independent of solutions at all other time points. If the circuit response for time t_o is all that is required this response can be calculated efficiently without calculating the response for any other values of time. Using inverse FFT techniques the entire waveform must be calculated.

IV. Examples

To illustrate the advantage of the proposed approach consider the case of a single lossless transmission line(Fig. 1) with an ideal input source at one end and a zero termination at the other end. The total inductance L and capacitance C are assumed to be $L = C = 1$. This is a simple circuit which may be solved directly. For a step input the closed-form solution for the current into the transmission line at time t is

$$i_s(t) = \mathcal{L}^{-1}\left[\frac{1}{s}\frac{1+e^{-2s}}{1-e^{-2s}}\right] = t + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi t)}{n} \quad (6)$$

where $\mathcal{L}^{-1}$ denotes inverse Laplace transform. For an input pulse of duration Δ, the current into the transmission line is given by

$$i_p(t) = i_s(t) - i_s(t-\Delta) \quad (7)$$

Obviously, the conductor's current is of infinite time-duration. Consequently, it is impossible to compute the current waveforms using inverse Fourier transform techniques. Using the proposed algorithm of numerical inversion of the Laplace transform current waveforms for this circuit can be computed. The results are compared with the closed-form solution in Figure 2.

A more complex example circuit is shown in Fig. 3. Using the method described in [6] this circuit would be divided into two transmission lines and a terminating network. The proposed MNA formulation uses a single matrix equation to represent the entire circuit.

It is impossible to match the transmission lines at all the terminal locations in the network of Fig. 3. The mismatches between the transmission lines and the terminal networks result in long transient periods. If FFT was used it would not be possible to obtain the section of response shown in Fig. 4 independent of the rest of the response. FFT requires that the analysis span a time interval over which all transients vanish.

V. Summary

A Laplace-domain modified nodal analysis method for calculating the time-domain response of a network containing lossy coupled transmission lines was described. The formulation of the network equations is based on a Laplace-domain admittance "stamp" for the transmission line. The transmission line stamp can be used to formulate equations representing arbitrarily complex networks of transmission lines and interconnect. These equations can be solved with $s = j\omega$ to get the frequency-domain response of the network.

Numerical inversion of the Laplace transform allows the time-domain response to be calculated directly from Laplace-domain equations. This method is an alternative to calculating the frequency-domain response and using FFT to obtain the time-domain response. The inversion technique is equivalent to high order, numerically stable integration methods. With numerical inversion, points in the time-domain response can be calculated independently from each other. This independence can be exploited when the entire time response is not required.

The proposed technique is particularly useful compared to the FFT approach in cases where the response of the line exceeds many transit times. The time-domain independence of the solution was exploited by an efficient calculation of the propagation delay of the network. In addition, the numerical inversion method can efficiently be used to calculate the initial transients in a mismatched coupled system where reflections result in very long response times.

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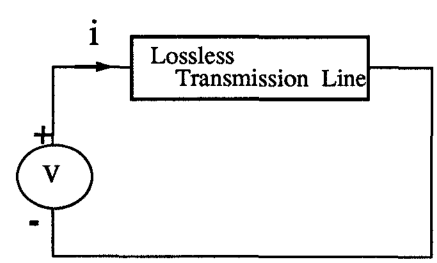

Figure 1: A lossless transmission line with ideal input source and zero termination.

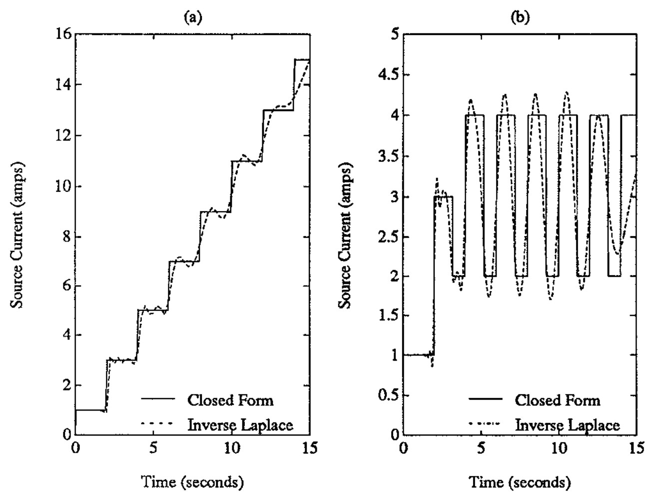

Figure 2: Shorted lossless transmission line: (a) Step Response, (b) Pulse Response

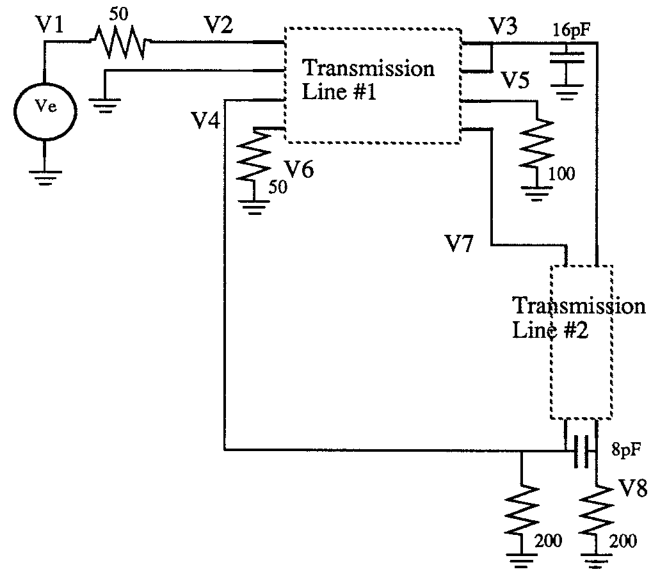

Figure 3: Two multiconductor transmission lines with terminal networks.

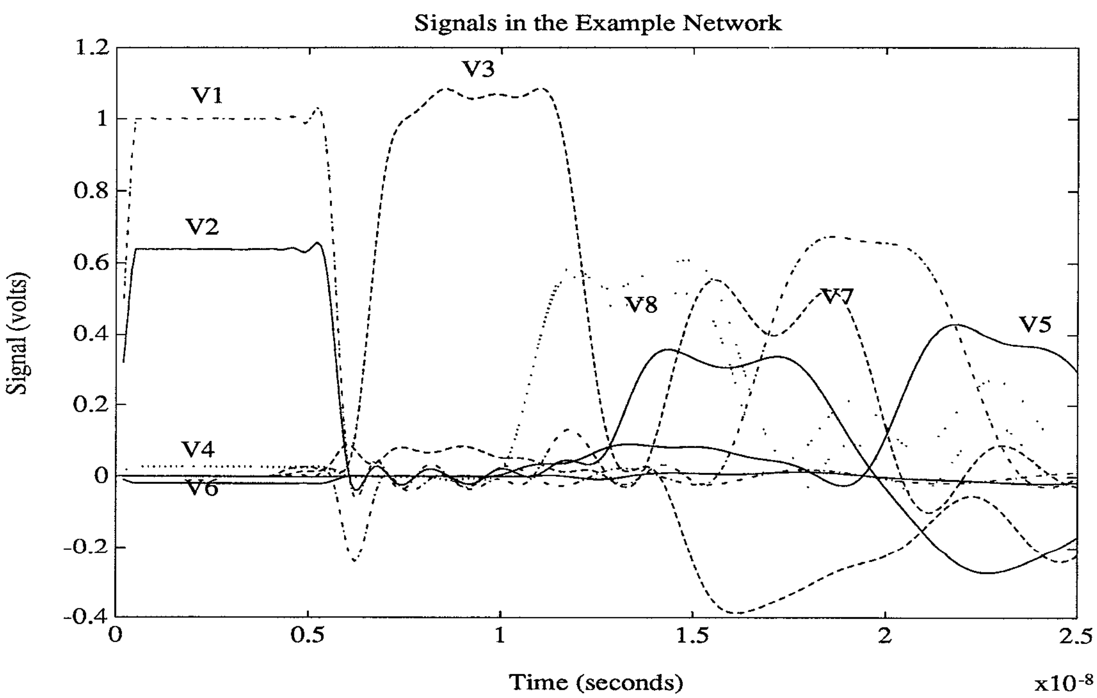

Figure 4: Response of the circuit shown in Fig. 3 computed using numerical inversion of the Laplace transform. The input V_e is shown as waveform $V1$.